Energy budget-based backscatter in a shallow water model driven by double gyre wind forcing

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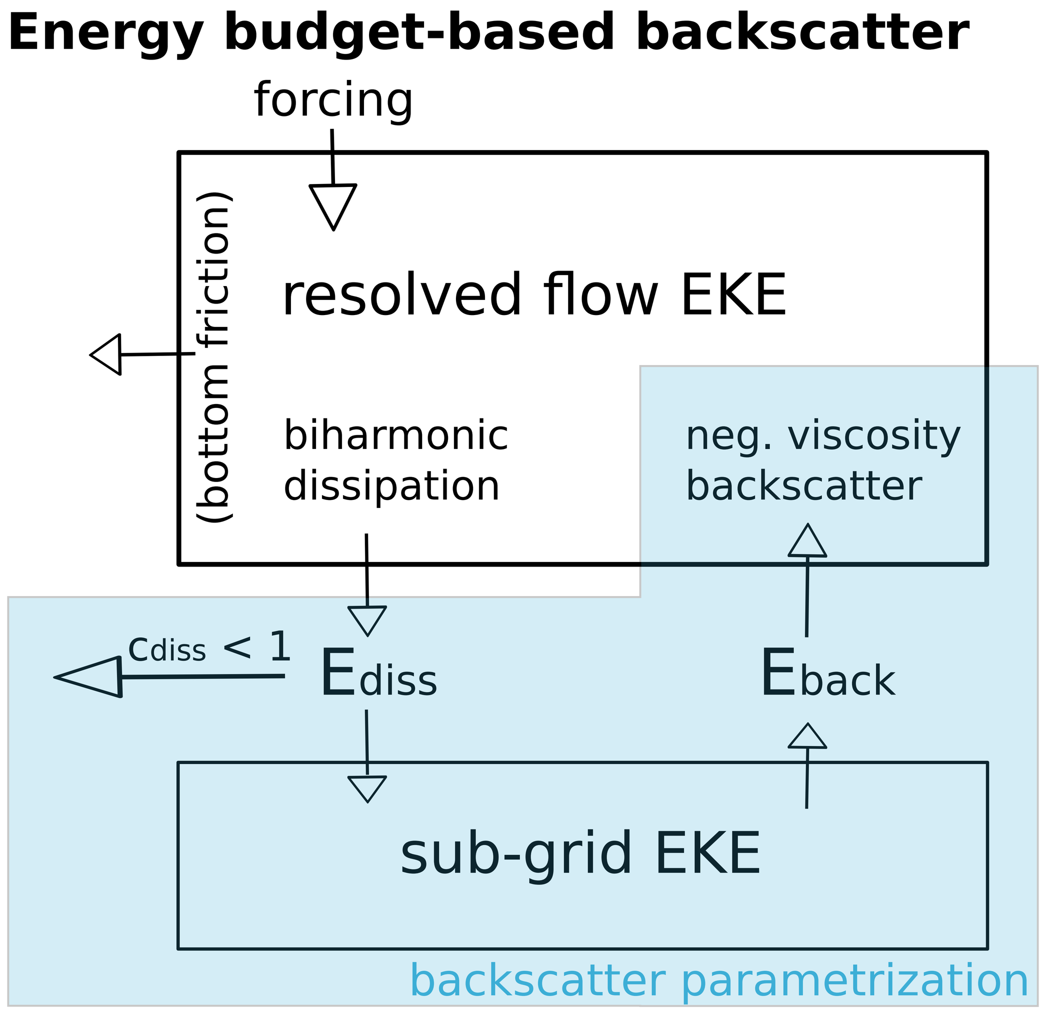
Abstract

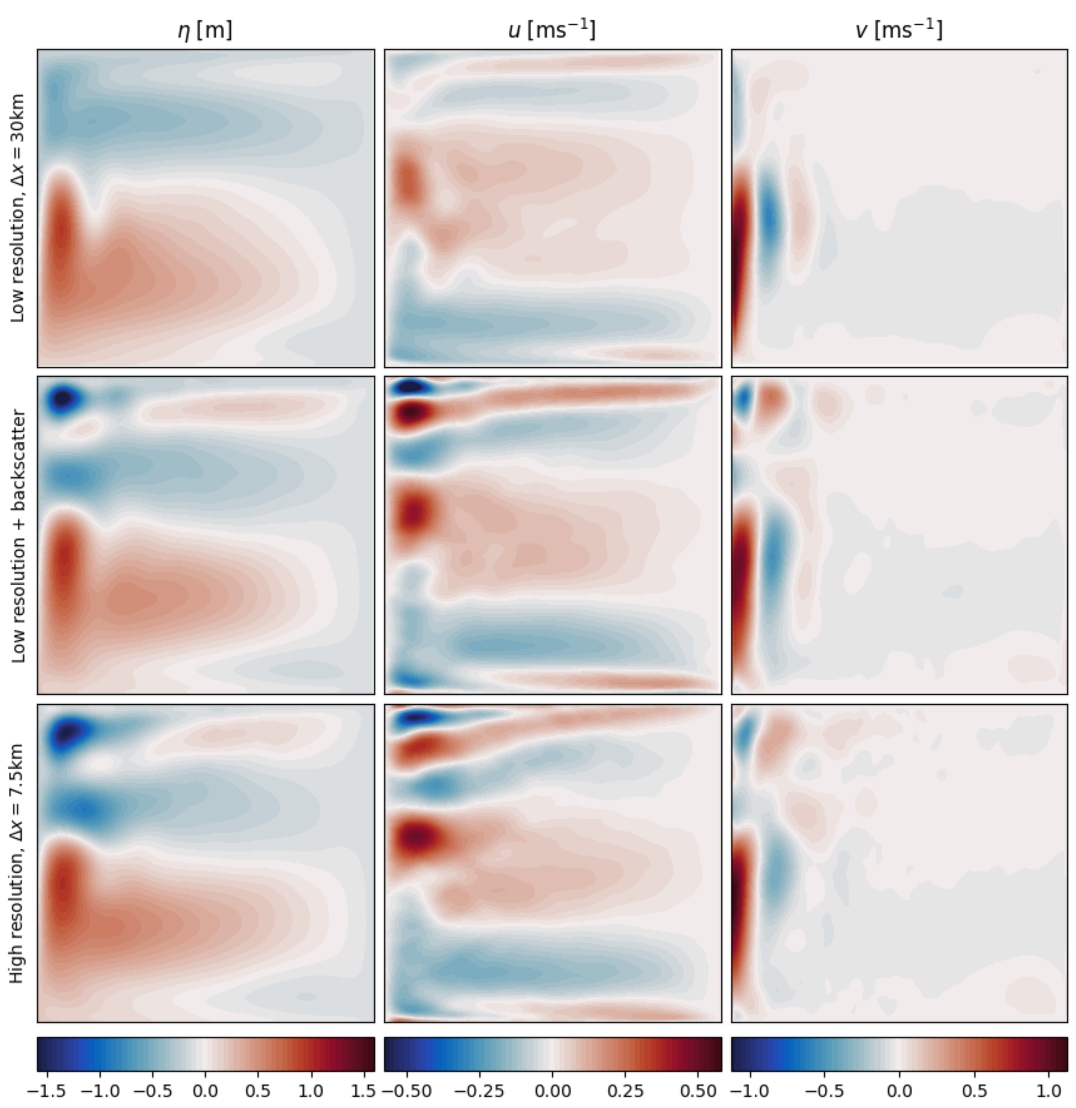
The parametrization of sub-grid scale processes is one of the key challenges towards improved numerical simulations of atmospheric and oceanic circulation. Numerical weather prediction models as well as climate models would benefit from more sophisticated turbulence closures that allow for less spurious dissipation at the grid-scale and consequently higher and more realistic levels of eddy kinetic energy (EKE). Recent studies[1,2] propose to use a hyperviscous closure in combination with an additional deterministic forcing term acting as negative viscosity to represent backscatter of energy from the unresolved scales. Here, we apply the parametrization to a shallow water model driven by double gyre wind forcing with no-slip boundary conditions and provide evidence for its general application.

Energy budget-based backscatter

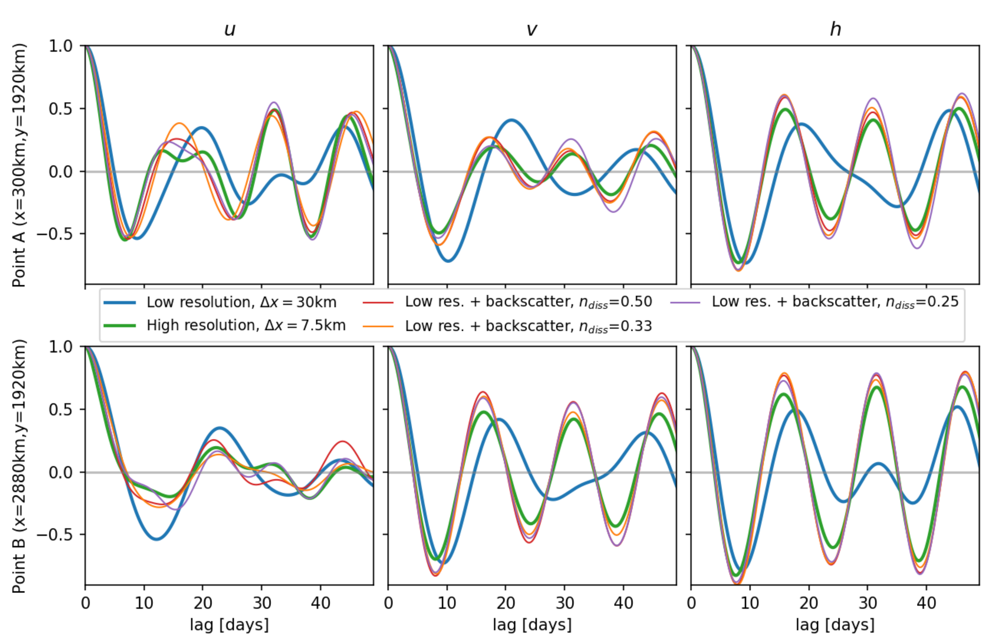


Energy budget-based backscatter schematically: The energy being dissipated by biharmonic viscosity is partly captured in the sub-grid EKE prognostic variable and partly removed where $c_{diss} < 1$. Based on the local levels of sub-grid EKE, a negative Laplacian viscosity reinjects energy at larger scales to recycle some of the spurious dissipation at the grid-scale.

Improvements on the mean state



Improvements on time scales



Autocorrelaion function of the prognostic variables. Point A is 300km away from the coast in the western boundary current detachment region. Point B is in the central eastern part of the domain.

Formulation: Shallow water model with backscatter

The sub-grid EKE is introduced as an additional prognostic variable[3] e, that is fed by dissipation at the grid scale, and enables recycling of EKE via the backscatter term (ξ_x, ξ_y) at larger scales

Shallow water model

$$\partial_t u + u\partial_x u + v\partial_y u - fv = -g\partial_x \eta + \frac{F_x}{\rho h} + M_x + \xi_x$$
$$\partial_t v + u\partial_x v + v\partial_y v + fu = -g\partial_y \eta + M_y + \xi_y$$
$$\partial_t \eta + \partial_x(uh) + \partial_y(vh) = 0$$
$$\partial_t e = -\dot{E}_{\text{diss}} + \dot{E}_{\text{back}} + h^{-1}\nabla \cdot \nu_e h \nabla e$$

+ backscatter parametrization

with lateral mixing of momentum with a biharmonic stress tensor $\mathbf{S}^*$

$$(M_x, M_y) = \nu_B h^{-1} \nabla \cdot h\mathbf{S}^* \approx \nu_B \nabla^4 \mathbf{u}$$

and the dissipated, backscattered energies

$$\dot{E}_{\text{diss}} = c_{\text{diss}} \nu_B h \nabla\mathbf{u} \cdot \mathbf{S}^*$$
$$\dot{E}_{\text{back}} = \nu_{\text{back}} h \nabla\mathbf{u} \cdot \mathbf{S}$$

Backscatter terms as Laplacian viscosity harmonic stress tensor $\mathbf{S}$

$$(\xi_x, \xi_y) = h^{-1}\nabla \cdot \nu_{\text{back}} h \mathbf{S}$$

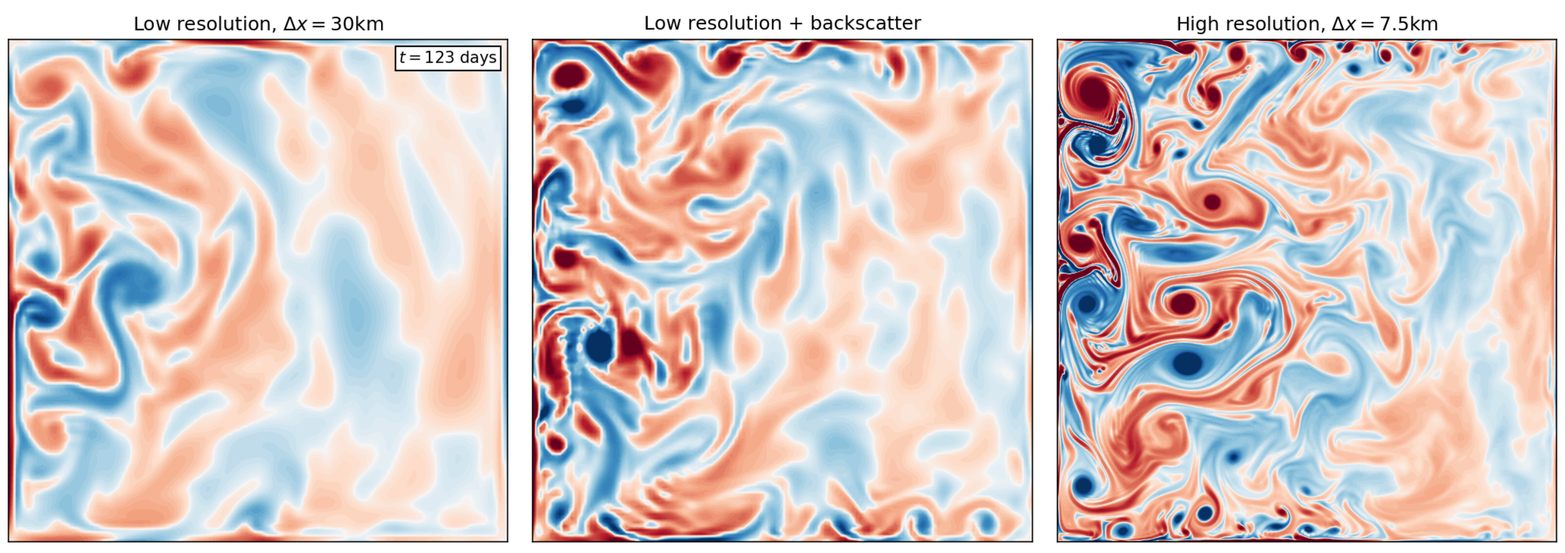
additional energy sink for high Rossby-number flow

$$c_{\text{diss}} = \frac{1}{(1+R_o)^{n_{\text{diss}}}}$$

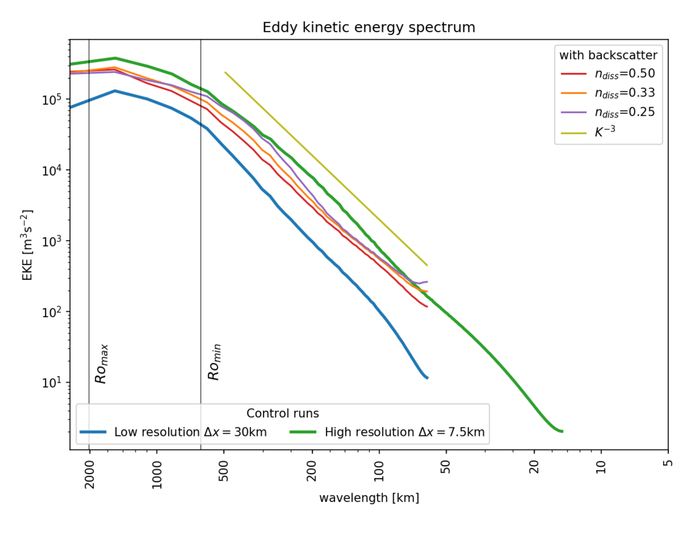
to reduce numerical noise at the boundary, n_{diss} is the remaining parameter for tuning.

negative viscosity coefficient based on subgrid-EKE e

$$\nu_{\text{back}} = -c_{\text{back}} \Delta x \sqrt{\max(2\frac{e}{h}, 0)}$$

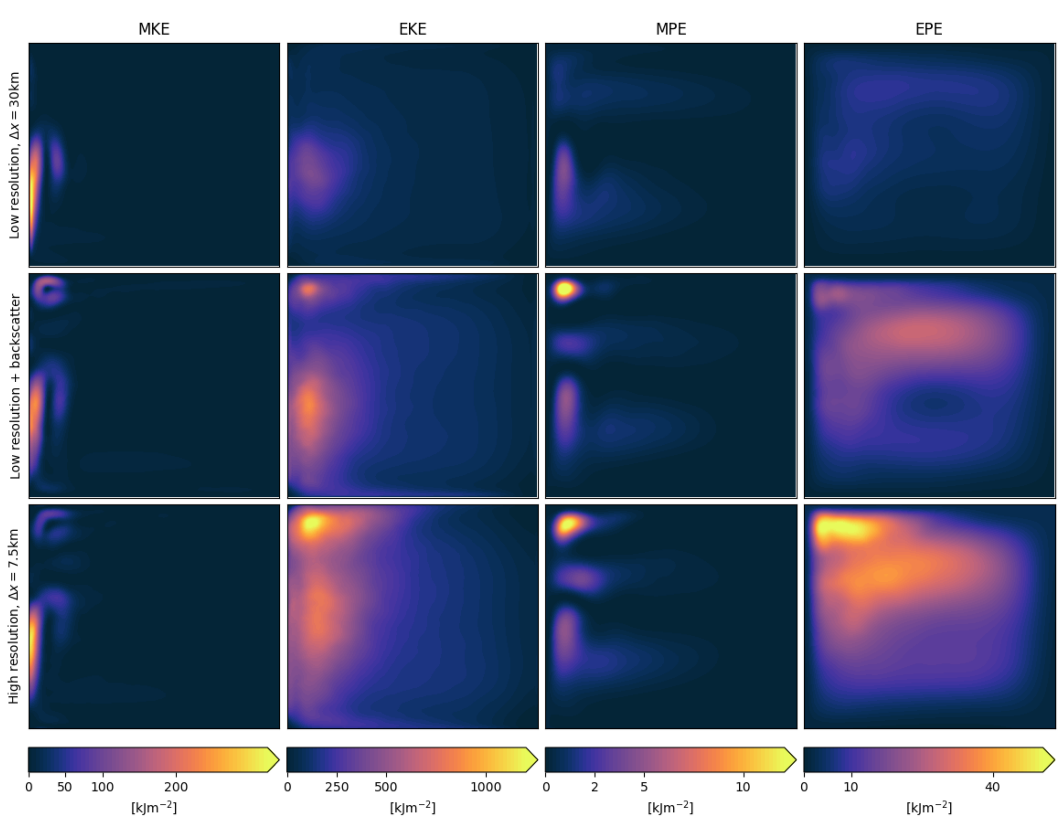


Snapshots of relative vorticity showing the increased eddy activity at the boundary introduced by the backscatter as most of the dissipation occurs at the boundary due to no-slip boundary conditions.



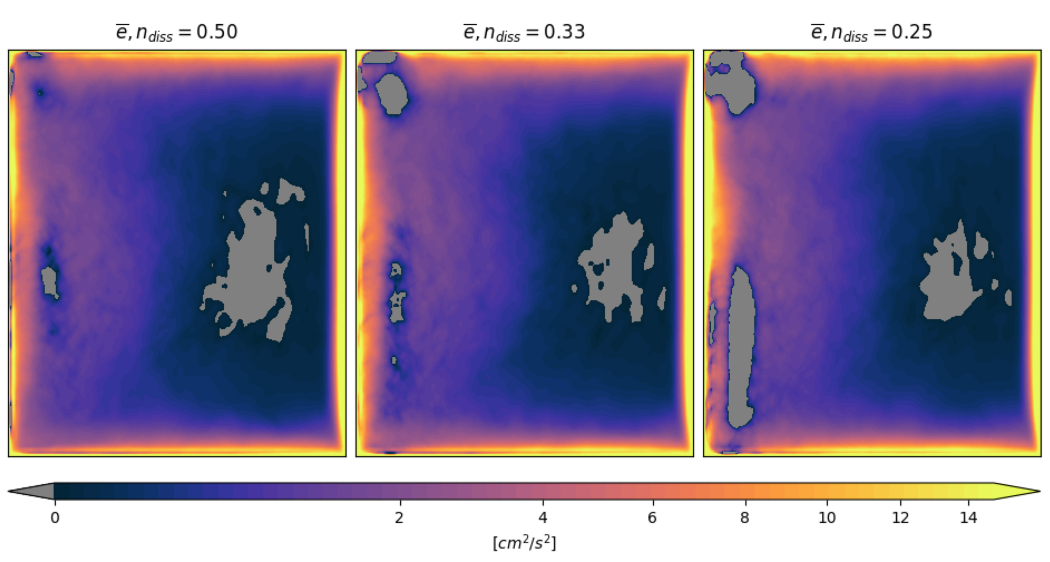
EKE starts to pile up at the grid scale once the backscatter is too strong (n_{diss} = 0.25).

Improvements on variability



Mean kinetic energy (MKE), EKE, mean potential energy (MPE) and eddy potential energy (EPE) are improved towards the high resolution truth.

Sub-grid eddy kinetic energy



Temporal mean of the sub-grid EKE variable for weak (n_{diss} = 0.50), moderate (n_{diss} = 0.33) and strong backscatter (n_{diss} = 0.25). No-slip boundary conditions are responsible for a majority of dissipation, hence the backscatter parametrization tends to reinject energy at the boundary.

Discussion

The energy-budget based backscatter parametrization effectively lowers the viscosity but keeps the numerical model stable by an artifical upscale transfer of energy, that results from the combination of biharmonic diffusion (i.e. the lateral mixing of momentum term) and harmonic anti-diffusion (i.e. backscatter term). The parametrization tends to accelerate eddies, that would otherwise be dissipated. We realize that increasing the amount of kinetic energy in the low resolution model compared to the high resolution truth yields a better simulation with respect to a variety of statistical measures but also pushes the model towards the edge of numerical stability. In a shallow water model with bottom friction the improvements of the backscatter formulation are assumed to be even greater, due to lowered levels of kinetic energy.

Using the backscatter formulation as presented here increases the computational cost by a factor of 1.5 compared to a factor of at least 16 by quadrupling the resolution. A wider application of energy-budget based backscatter is therefore promising in order to remove model biases as well as increasing the quality of predictions.

References

[1] M F Jansen et al., Ocean Modelling 94, 15-26 (2015)
[2] M F Jansen and I M Held, Ocean Modelling 80, 36-48 (2014)
[3] C Eden and R J Greatbatch, Ocean Modelling 20, 223-239 (2008)

Model code written in Python

github.com/milankl/swm

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